II. Solution by P. S. BERG, B.Sc., Superintendent of Schools, Larimore, N. D.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; ELMER SCHUYLER, High Bridge, N. J.; W. H. WILSON, Professor of Mathematics, Geneva College, Beaver Falls, Pa.; and G. B. M. ZERR, A. M., Ph.D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $OP=p$, $OC=a$, $OA=b$, $OB=c$, $OI=l$, $IH=m$, $PH=n$.

The triangles (right angled) OPA and OPI are similar, having $\angle POA$ in common.

$\therefore$ $p/b=l/p$, or $p^2/b^2=l^2/p^2$, or $p^4/b^2=l^2$.

Similarly, $p^4/a^2=n^2$, $p^4/c^2=m^2$.

$\therefore$ $p^4/a^2+p^4/b^2+p^4/c^2=n^2+l^2+m^2=p^2$.

$\therefore$ $1/a^2+1/b^2+1/c^2=1/p^2$.

Also solved by *J. SCHEFFER*, and *E. D. SCALES*.

111. Proposed by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Given that the area of a triangle is equal to half the product of two sides and the sine of the included angle, prove that $\sin(x+y)=\sin x\cos y+\cos x\sin y$.

I. Solutfon by W. F. BRADBURY, A. M., Head Master, Cambridge Latin School, Cambridge, Mass., and B. F. YANNEY, A. M., Professsor of Mathematics, Mount Union College, Alliance, O.

Given area $\triangle=\frac{1}{2}ac\sin B$.... .(1).

$\sin B=\sin[180^\circ-(A+C)]=\sin(A+C)$.

$\therefore$ Area $\triangle=\frac{1}{2}ac[\sin(A+C)]$......(2).

Draw BD perpendicular to AC.

Area $\triangle=\frac{1}{2}BD(AD+DC)$.

But $BD=c\sin A=a\sin C$, and $AD=c\cos A$, and $DC=a\cos C$.

$$\therefore \text{Area } \triangle=\tfrac{1}{2}\begin{matrix} c\sin A \\ \text{or} \\ a\sin C\end{matrix}(c\cos A+a\cos C)=\tfrac{1}{2}(ac\sin C\cos A+ac\sin A\cos C)\ldots(3).$$

Putting (2)=(3), $\frac{1}{2}ac[\sin(A+C)]=\frac{1}{2}ac(\sin A\cos C+\cos A\sin C)$ or $\sin(A+C)=\sin A\cos C+\cos A\sin C$.

II. Solution by J. OWEN MAHONEY, B. E., Professor of Mathematics and Science, Carthage High School, Carthage, Tex.; J. W. YOUNG, Columbus, O.; J. SCHEFFER, A. M., Hagerstown, Md.; E. L. SHERWOOD, A. M., Professor of Mathematics, Whiteworth College, Miss.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; B. F. SINE, Principal of Normal School, Capon Bridge, W. Va.; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; JOHN MACHNIE, A. M., Professor of Latin, University of North Dakota; H. F. STRATTON, Student in Heidelberg University, Tiffin, O.; J. C. NAGLE, C. E., Professor of Civil Engineering and Physics in the Agricultural and Mechanical College of Texas, College Station, Tex.; CHARLES C. CROSS, Libertytown, Md.; and ELMER SCHUYLER, High Bridge, N. J.

PROOF. Consider the triangle ACB.

The area of $ACB=\frac{1}{2}ac\sin B=\frac{1}{2}ac\sin(A+C)=\frac{1}{2}hb=\frac{1}{2}h(AD+DC)$, or

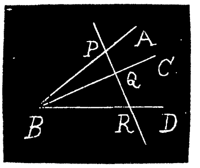
$$\sin(A+C)=\frac{h}{a}.\frac{AD}{c}+\frac{h}{c}.\frac{DC}{a}$$

$$=\sin C\cos A+\sin A\cos C.$$

$$\therefore \sin(x+y)=\sin x\cos y+\cos x\sin y.$$

III. Solution by the PROPOSER.

Let ABC and CBD be the two angles x and y. Draw any line perpendicular to BC meeting AB, BC, BD at P, Q, R.



Then $\triangle BPR = \triangle BPQ + \triangle BQR$.

$2\triangle BPR = BP\times BR\times \sin PBR = BP.BR.\sin(x+y)$.

$2\triangle BPR = BP\times BQ\times \sin PBQ = BP.BQ.\sin x$.

$2\triangle BQR = BQ.RB.\sin y$.

Hence $BP.BR\sin(x+y) = BP.BQ\sin x + BQ.BR\sin y$.

Dividing by $BP.BR$, $\sin(x+y) = \frac{BQ}{BR}\sin x + \frac{BQ}{BP}\sin y = \cos y\sin x + \cos x\sin y$.

CALCULUS.

84. Proposed by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, Ohio.

Find the equation of the curve upon which a given ellipse must roll in order that one of its foci may describe a straight line.

Solution by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Let θ be the angle between the major axis of the ellipse and the radius vector from the focus to the point of contact; r the length of this radius, a, b, semi-axes, e eccentricity. Then

$$r = \frac{a(1-e^2)}{1-e\cos\theta}.$$

Let the axis of x be taken parallel to the given line. Then since the point of contact is the instantaneous center, the radius vector will always be perpendicular to the axis of x, and hence $r+y=a(1-e)$, y being the ordinate of a point on the required curve. We have also

$$r\frac{d\theta}{dr} = -\frac{dx}{dy}.$$

Eliminating r and θ, we find

$$y - \sqrt{b^2\left(\frac{dy}{dx}\right)^2 + a^2} = ae.$$

Whence $\frac{ae-y}{b} = \sin\left(\frac{x}{b} + \sin^{-1}\frac{ae}{b}\right)$, or $y' = b\sin\left(\frac{x'}{b}\right)$.

86. Proposed by WALTER H. DRANE, Graduate Student, Harvard University, 65 Hammond Street, Cambridge, Mass.

Prove that the curve whose normal equals its radius of curvature drawn in an opposite direction, is the catenary $y = c\cosh(x/c)$.